The one-body and two-body density matrices of finite nuclei with an appropriate treatment of the center-of-mass motion[⋆]

A. Shebeko[1,a], P. Papakonstantinou[2,b], and E. Mavrommatis[3,c]

[1] NCS "Kharkov Institute of Physics and Technology", Academicheskaya Str. 1, 61108 Kharkov, Ukraine
[2] Institute of Nuclear Physics, T.U. Darmstadt, Schlossgartenstr. 9, D-64289 Darmstadt, Germany
[3] University of Athens Physics Department, Nuclear and Particle Physics Division, Panepistimiopoli, Ilissia, GR-157 71 Athens, Greece

Abstract. The one-body and two-body density matrices in coordinate space and their Fourier transforms in momentum space are studied for a nucleus (a nonrelativistic, self-bound finite system). Unlike the usual procedure, suitable for infinite or externally bound systems, they are determined as expectation values of appropriate intrinsic operators, dependent on the relative coordinates and momenta (Jacobi variables) and acting on intrinsic wave functions of nuclear states. Thus, translational invariance (TI) is respected. When handling such intrinsic quantities, we use an algebraic technique based upon the Cartesian representation, in which the coordinate and momentum operators are linear combinations of the creation and annihilation operators $\hat{\mathbf{a}}^{\dagger}$ and $\hat{\mathbf{a}}$ for oscillator quanta. Each of the relevant multiplicative operators can then be reduced to the form: one exponential of the set $\{\hat{\mathbf{a}}^{\dagger}\}$ times another exponential of the set $\{\hat{\mathbf{a}}\}$. In the course of such a normal-ordering procedure we offer a fresh look at the appearance of "Tassie-Barker" factors, and point out other model-independent results. The intrinsic wave function of the nucleus in its ground state is constructed from a nontranslationally-invariant (nTI) one via existing projection techniques. As an illustration, the one-body and two-body momentum distributions (MDs) for the ^{4}He nucleus are calculated with the Slater determinant of the harmonic-oscillator model as the trial, nTI wave function. We find that the TI introduces quite important effects in the MDs.

1 Introduction

In the last few years the interest in the study of nuclei from both experimental and theoretical point of view has shifted from the investigation of one-body quantities (*e.g.*, the elastic form factor $F(\mathbf{q})$ and the momentum distribution $\eta(\mathbf{p})$) towards the investigation of two-body quantities, with the aim of revealing more direct information on the dynamical correlations between the nucleons (short-range (SRC) and tensor). The two-body quantities are connected to the two-body density matrix (2DM) in coordinate or momentum space as are the one-body quan-

tities connected to the one-body density matrix (1DM). The 2DM, besides being interesting in itself, allows the calculation of the expectation value of any two-body operator [1]. In addition to the 2DM $\rho^{[2]}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_{1'}, \mathbf{r}_{2'})$, we will also consider the two-body momentum distribution (TBMD) $\eta^{[2]}(\mathbf{p}, \mathbf{k})$, which is the Fourier transform of the $\rho^{[2]}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_{1'}, \mathbf{r}_{2'})$ in the variables $\mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{r}_{1'} - \mathbf{r}_{2'}$ and is connected to the two-nucleon spectral function $S(\mathbf{p}, \mathbf{k}; E)$ via integration with respect to the energy E.

A prominent role towards the experimental investigation of the 2DM and related quantities is played by the study of the electromagnetically induced 2-nucleon emission (γ, NN), $(\mathrm{e}, \mathrm{e'NN})$ which can be carried out with high accuracy in photon facilities (Elsa, MAMI) and electron accelerators with high energy, 100% duty-cycle beams (Jefferson Lab, MAMI). Past, present and near future experiments provide these useful data [2–6]. Theoretical methods to analyze the mechanisms of these reactions and to calculate the relevant nuclear two-body proper-

[a] e-mail: shebeko@kipt.kharkov.ua
[b] e-mail: panagiota.papakonstantinou@physik.tu-darmstadt.de
[c] e-mail: emavrom@phys.uoa.gr

ties are under continuous development. In particular, for the case of finite nuclei the generalized momentum distribution $\eta(\mathbf{p}, \mathbf{Q})$ [7,8], the two-body momentum distribution $\eta^{[2]}(\mathbf{p}, \mathbf{k})$ and other two-body distributions have been studied for $Z = N$, ℓ-closed nuclei, as well as the two-body density matrix for the nuclei ^{4}He [9], ^{16}O and ^{40}Ca [10] and the two-nucleon spectral function $S(\mathbf{p}, \mathbf{k}; E)$ for the nucleus ^{16}O (see [11] and references therein).

One of the theoretical issues still under discussion is the proper consideration of the requirement of translational invariance and therefore the conservation of the total momentum of the system. The wave functions which have been used in the independent-particle shell model and in theories which take also dynamical correlations into account (e.g., Brueckner-Hartree-Fock, Variational Monte Carlo) satisfy the Pauli principle but not the translational invariance. As a consequence, they contain spurious components which result from the motion of the Center of Mass (CM) in a non-free state. Effects from these (also known as CM correlations) are found in the calculation of almost every observable and make impossible the extraction of information for the intrinsic properties of nuclei directly from the experimental data. In addition, there is ambiguity in the proper definition of translationally invariant operators which correspond to the different physical quantities.

Many efforts have been made towards the solution of the CM problem. In some of them the treatment of the CM motion is built right into the theory one is using [12–14]. In the majority of the efforts the restoration of translational invariance is attempted after the wave functions have been developed. One such approach consisted of adding intuitively the three extra degrees of freedom of the CM to the $3A$ internal coordinates. Work along these lines using configurations from the harmonic-oscillator model (HOM) has been carried out by Tassie and Barker [15] and others. Most of the other approaches use projection techniques to define suitable intrinsic wave functions with coordinates referred to the CM. The pioneering works along this direction were made in the '50s by Gartenhaus and Schwartz (GS) [16] and Peierls and J. Yoccoz (PY) [17], with violations of the Galilei invariance (GI), followed by Ernst, Shakin and Thaler (EST) [18,19], with their critique on the GS transformation, Vincent [20], Shebeko et al. [21,22] and others in the '70s [13,23]. Projection techniques have been proposed by Schmid and Grümmer [24,25] at the beginning of the '90s and rather recently by Schmid and collaborators using harmonic-oscillator [26] and spherical Hartree-Fock configurations [27]. We should add that Mihaila and Heisenberg have worked out the problem of CM corrections by expanding them as many-body operators [28]. It seems that if the wave function is very nearly factorable into a center-of-mass and an intrinsic component, all the approaches to treat the CM problem are equivalent, provided that the translationally invariant operators are used. Also, all the approaches can be carried out rather simply for the independent particle shell model with harmonic-oscillator potential.

The consideration of CM effects with one or more of the above-mentioned methods have mostly addressed the light nucleus ^{4}He and single-particle quantities: the kinetic energy, the single-particle energies, the one-body density matrix, the matter and charge density, the elastic form factor, the dynamic structure factor, the momentum distribution and occupation probabilities, the one-body spectral function, the single-particle overlap function, etc. There are few calculations for other light-medium nuclei such as ^{12}C, ^{15}N, ^{16}O, ^{40}Ca. The consideration of CM correlations in two-body quantities has been limited so far, to the best of our knowledge to the potential energy.

In this paper, we will start with the evaluation of the one-body density matrix in coordinate space and related one-body momentum distribution (OBMD). As mentioned above, CM effects have been considered in the calculation of 1DM and OBMD before. By starting with such evaluation we want to present our method and technicalities. We then proceed to evaluate the two-body density matrix and two-body momentum distribution. We will take into account the CM correlations by using the EST prescription or fixed-CM approximation to construct the intrinsic wave function. We also use a specific prescription for defining the corresponding intrinsic operators. As mentioned before, the EST method has been introduced in refs. [18,19]. Subsequently, it has been used in the calculation of the elastic and dynamic form factor and the momentum distribution of ^{4}He [22,29]. The EST intrinsic many-body wave function is constructed from a translationally non-invariant one by projecting onto an eigenstate of total momentum using a non-unitary operator which fixes the CM coordinate R to be equal to zero. This transformation has certain advantages compared to GS transformation [19] (in particular, it ensures a correct behavior under Galilean transformation). It has turned out [19] (see also [22]) that the GS transformation can be reduced to the EST projection procedure. As for the relevant one- and two-body intrinsic operators, they are defined by replacing the coordinates and momenta by relative ones (Jacobi variables). Unlike the definitions of the overlap integrals with intrinsic wave functions used in ref. [30], we are dealing with the expectation values of the intrinsic operators as they occur under the treatment of the aforementioned quantities. Subsequently, we present the way for calculating the matrix elements defined by the above intrinsic wave functions and operators using an algebraic technique introduced in ref. [21] and based upon the Cartesian representation of the coordinate and momentum operators in terms of linear combinations of the creation and annihilation vector operators ($\hat{\mathbf{a}}^\dagger$ and $\hat{\mathbf{a}}$, respectively) for oscillator quanta in the three different space directions. With this technique, one avoids to deal with difficult multiple integrals and therefore it seems to be the technique of choice in the case of systems with large number of bodies. The Cartesian representation is particularly convenient in the case of wave functions constructed with Slater determinants. The application of the above to the evaluation of the OBMD leads to the derivation of the Tassie-Barker factor [15] ($\exp[r_0^2 q^2/4A]$, r_0 oscillator

length parameter entering into the definition of $\hat{a}^\dagger$ and $\hat{a}$ —see sect. 2) in a model-independent way. In addition, the evaluation of both OBMD and TBMD leads to other model-independent results.

Next, the intrinsic OBMD and TBMD are evaluated in the independent particle shell model with harmonic-oscillator wave functions. Such a model leads to compact analytical expressions. Moreover, it is expected that the results derived with its use will be close to the ones that will be obtained with the use of more realistic single-particle wave functions (Woods-Saxon or Hartree-Fock), since the quantities under study are not defined in terms of the asymptotic behavior of the wave function for large r. The latter is wrong in the case of the wave function in the HOM.

In this work, the above evaluation is carried out for the intrinsic OBMD and TBMD of the nucleus ^{4}He. It is expected that the CM effects are more pronounced for light nuclei. In addition, due to its high central density (almost 3 times nuclear matter density) the nucleus ^{4}He is a particularly appropriate system to search for the origin of SRC. The evaluation of the intrinsic OBMD of ^{4}He has appeared before in the case of HOM [22] as well as other single-particle models [29]. The (not intrinsic) TBMD of ^{4}He has been studied in refs. [11,31] by including in the HOM Jastrow-type correlations via the lowest term of the so-called low-order approximation [32], but ignoring CM correlations. A comparison of the present results with those of refs. [11,31] will reveal the relative importance of the CM and SRC effects in the same nucleus. The effect of the different correlations is estimated by introducing the quantity $\eta^{[2]}(\mathbf{p}, \mathbf{k})/\eta(\mathbf{p})\eta(\mathbf{k})$. We find that the CM correction reduces the width of OBMD and TBMD and in the case of the TBMD introduces a dependence on the angle between $\mathbf{p}$ and $\mathbf{k}$, a shift of its peak in favor of opposite momenta and significant deviations for large values of p and k and angles close to $180°$. This last effect is also found when SRC are considered. Up to now, the TBMD of ^{3}He has been experimentally studied at Jefferson Lab [5].

The paper is organized as follows. In sect. 2, we present the general formalism of constructing appropriate wave functions that respect translational invariance using the Ernst-Shakin-Thaler prescription and describe the evaluation of matrix elements of intrinsic operators using the Cartesian representation. In sect. 3 the definitions of the relevant operators for the intrinsic quantities under study, namely the one-body quantities 1DM, form factor and OBMD and the two-body quantities 2DM and TBMD, are introduced in terms of the relative coordinates and momenta (Jacobi variables). In sect. 4 the above quantities are evaluated using the Cartesian representation. In sect. 5 by considering specifically the independent-particle shell model with harmonic-oscillator wave functions, results are derived and discussed for the OBMD and TBMD of the nucleus ^{4}He. Finally, in sect. 6 a summary of the results and hints for possible further work are given.

2 Constructing intrinsic wave functions and matrix elements. The Cartesian representation

Let us consider a nonrelativistic system composed of A particles (nucleons). The coordinate (momentum) vector of the α-th particle will be denoted by $\mathbf{r}_\alpha$ ($\mathbf{p}_\alpha$). Occasionally, we will use the generic symbol α, which may include spin and/or isospin degrees of freedom, but in most cases we will suppress these degrees of freedom for the sake of simplicity.

In principle, the eigenvectors of the total Hamiltonian $\hat{H}$ of the system $|\Psi_\mathbf{P}\rangle$, which belong to the eigenvalue $\mathbf{P}$ of the total momentum operator $\hat{\mathbf{P}}$, can be written as the product

$$|\Psi_\mathbf{P}\rangle = |\mathbf{P}) \, |\Psi_\text{int}\rangle. \tag{1}$$

Following ref. [18], the bracket $|\,)$ is used to represent a vector in the space of the center-of-mass coordinates, so that $\hat{\mathbf{P}}|\mathbf{P}) = \mathbf{P}|\mathbf{P})$. A ket (bra) with an index $|\cdots\rangle_\alpha$ ($_\alpha\langle\cdots|$) will refer to the state of the α-th particle. The intrinsic wave function Ψ_int depends upon the $A-1$ independent intrinsic variables. These may be expressed in terms of the Jacobi coordinates

$$\boldsymbol{\xi}_\alpha = \mathbf{r}_{\alpha+1} - \frac{1}{\alpha}\sum_{\beta=1}^{\alpha}\mathbf{r}_\beta \qquad (\alpha = 1, 2, \ldots, A-1) \tag{2}$$

or the corresponding canonically conjugate momenta

$$\boldsymbol{\eta}_\alpha = \frac{1}{\alpha+1}\left(\alpha\mathbf{p}_{\alpha+1} - \sum_{\beta=1}^{\alpha}\mathbf{p}_\beta\right) \qquad (\alpha = 1, 2, \ldots, A-1). \tag{3}$$

The wave function $\Psi_\mathbf{P}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)$ in the coordinate representation satisfies the requirement of translational invariance,

$$\Psi_\mathbf{P}(\mathbf{r}_1+\mathbf{a}, \mathbf{r}_2+\mathbf{a}, \ldots, \mathbf{r}_A+\mathbf{a}) = \exp(i\mathbf{P}\cdot\mathbf{a})\Psi_\mathbf{P}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A), \tag{4}$$

for any arbitrary displacement $\mathbf{a}$.

When describing scattering processes, it is convenient to consider the initial target state $|0\rangle$ as a $\mathbf{P}$-packet,

$$|0\rangle = \int |\Psi_\mathbf{P}\rangle d\mathbf{P}\langle\Psi_\mathbf{P}|0\rangle \equiv \int c(\mathbf{P})|\Psi_\mathbf{P}\rangle \, d^3P \tag{5}$$

(see also [33], Chapt. XI), with the normalization condition

$$\langle 0 \mid 0 \rangle = \int |c(\mathbf{P})|^2 \, d^3P = 1. \tag{6}$$

Being the exact $\hat{H}$-eigenvectors, the states $|\Psi_\mathbf{P}\rangle$ belong simultaneously to the set of eigenvectors of the total momentum operator $\hat{\mathbf{P}}$ with eigenvalues $\mathbf{P}$ close to a given value $\mathbf{P}_t$, *e.g.*, $\mathbf{P}_t = 0$. The final state of the recoiling nucleus is written in the form $|\Psi_{\mathbf{P}'}\rangle$. Evidently, the wavepacket $|0\rangle$ is not translationally invariant. However, this shortcoming can be corrected by letting the width of

the packet Δ go to zero at the end of the calculations, *i.e.*, assuming that

$$\lim_{\Delta \to 0} \int |c(\mathbf{P})|^2 g(\mathbf{P}) \mathrm{d}^3 P = \int \delta(\mathbf{P} - \mathbf{P}_t) g(\mathbf{P}) \mathrm{d}\mathbf{P} = g(\mathbf{P}_t) \tag{7}$$

for an arbitrary function $g(\mathbf{P})$.

This prescription has a transparent physical meaning being adequate to many scattering situations. With its aid one can express the corresponding cross-sections in terms of intrinsic quantities. Let us consider, for instance, the elastic scattering of a particle from the nucleus. In the Plane Wave Impulse Approximation (PWIA) and neglecting the Fermi-motion effects, the cross-section of interest can be represented in the form

$$\sigma(\theta) = \lim_{\Delta \to 0} \int K(\mathbf{P}') \, \mathrm{d}^3 P' |\langle \Psi_{\mathbf{P}'} | \exp[i\mathbf{q} \cdot \hat{\mathbf{r}}_A] |0\rangle|^2, \quad (8)$$

where $\mathbf{q}$ is the momentum transfer, θ the scattering angle and $K(\mathbf{P}')$ the corresponding kinematical factor. By substituting $|0\rangle$ from eq. (5), using eq. (1) and writing $\hat{\mathbf{r}}_A = (\hat{\mathbf{r}}_A - \hat{\mathbf{R}}) + \hat{\mathbf{R}}$, we evaluate $\langle \Psi_{\mathbf{P}'} | \exp[i\mathbf{q} \cdot \hat{\mathbf{r}}_A] |0\rangle = \int \mathrm{d}^3 P c(\mathbf{P})(\mathbf{P}' | \exp(i\mathbf{q} \cdot \hat{\mathbf{R}}) |\mathbf{P}\rangle \langle \Psi_{\mathrm{int}} | \exp[i\mathbf{q} \cdot (\hat{\mathbf{r}}_A - \hat{\mathbf{R}})] |\Psi_{\mathrm{int}}\rangle = \int \mathrm{d}^3 P c(\mathbf{P}) \delta(\mathbf{P}' - \mathbf{P} - \mathbf{q}) F_{\mathrm{int}}(\mathbf{q})$, that leads to

$$\sigma(\theta) = K(\mathbf{P}_t + \mathbf{q}) \left| F_{\mathrm{int}}(\mathbf{q}) \right|^2,$$

where the elastic (intrinsic) form factor (FF) $F_{\mathrm{int}}(\mathbf{q})$ is determined by

$$F_{\mathrm{int}}(\mathbf{q}) = \langle \Psi_{\mathrm{int}} | \exp[i\mathbf{q} \cdot (\hat{\mathbf{r}}_A - \hat{\mathbf{R}})] |\Psi_{\mathrm{int}}\rangle \equiv \langle \Psi_{\mathrm{int}} | \hat{F}_{\mathrm{int}}(\mathbf{q}) |\Psi_{\mathrm{int}}\rangle. \tag{9}$$

Use has been made, on the one hand, of the fact that intrinsic and CM operators commute with each other and, on the other hand, of the condition (7). The $F_{\mathrm{int}}(\mathbf{q})$ is typical of the quantities of interest, *viz*, it is the expectation value of an operator that depends on intrinsic coordinates, namely $\hat{\mathbf{r}}_A - \hat{\mathbf{R}}$.

As we have mentioned in the introduction, our aim is to calculate expectation values of one- and two-body operators in the nuclear ground state (g.s.) taking into account the requirement for TI. We are interested, in particular, in intrinsic quantities which appear in analytical expressions describing various scattering cross-sections off a nucleus (in general, a finite system). Such quantities include, besides the elastic FF $F(\mathbf{q})$, the particle density $\rho(\mathbf{r})$, the dynamical FF $S(\mathbf{q}, \omega)$, the OBMD $\eta(\mathbf{p})$ which is often associated with the one-body spectral function $P(\mathbf{p}, E)$, and the TBMD $\eta^{[2]}(\mathbf{p}, \mathbf{k})$ that is related to the two-body spectral function $S(\mathbf{p}, \mathbf{k}; E)$.

At the initial stage of the calculations, the nuclear g.s. is represented by a $\mathbf{P}$-packet as introduced above. Then the main task is to construct the TI wave functions $|\Psi_{\mathbf{P}}\rangle$ in a tractable manner, so that the CM-motion separation can be achieved. It is important to properly define the quantities of interest in terms of intrinsic coordinates, as was already done for the FF in eq. (9). We will tackle this issue in the next section.

First of all, let us consider a Slater determinant,

$$|\mathrm{Det}\rangle = \frac{1}{\sqrt{A!}} \sum_{\hat{\mathcal{P}} \in S_A} \epsilon_{\mathcal{P}} \hat{\mathcal{P}}\{|\phi_{p_1}(1)\rangle |\phi_{p_2}(2)\rangle \cdots |\phi_{p_A}(A)\rangle\}, \tag{10}$$

as the total wave function $|\Psi_0\rangle$ for an approximate and convenient description of the nuclear g.s., in the framework of the independent-particle model or the Hartree-Fock approach. In eq. (10), $\epsilon_{\mathcal{P}}$ is the parity factor for the permutation $\hat{\mathcal{P}}$, ϕ_{p_α} are the occupied single-particle orbitals and the summation runs over all permutations of the symmetric group S_A.

The wave function (10) exemplifies wave functions which do not possess the property of TI, eq. (4). Obviously, any wave function that is constructed by acting on $|\mathrm{Det}\rangle$ with a two- or a three-body correlation operator (*e.g.*, a Jastrow correlation factor) will not be translationally invariant either. There are different ways to restore TI if one starts with a "bad" wave function $|\Psi\rangle$ such as $|\mathrm{Det}\rangle$ [17,18,25,26]. We shall employ the so-called "fixed-CM approximation", or EST prescription [18]. However, other projection recipes can be applied without essential changes —we will come back to this point at the end of sect. 4. Within the EST approach, the approximate complete wave function is determined by

$$|\Psi_{\mathbf{P}}^{\mathrm{EST}}\rangle = |\mathbf{P})|\Psi_{\mathrm{int}}^{\mathrm{EST}}\rangle \tag{11}$$

and the intrinsic unit-normalized wave function is given by

$$|\Psi_{\mathrm{int}}^{\mathrm{EST}}\rangle = (\mathbf{R} = 0|\Psi\rangle / [\langle \Psi | \delta(\hat{\mathbf{R}}) |\Psi\rangle]^{1/2}, \tag{12}$$

where $(\mathbf{R} = 0|$ is the eigenvector of the CM operator, $\hat{\mathbf{R}} = A^{-1} \sum_{\alpha=1}^{A} \hat{\mathbf{r}}_\alpha$. We have used the relation $|\mathbf{R} = \mathbf{X})(\mathbf{R} = \mathbf{X}| = \delta(\hat{\mathbf{R}} - \mathbf{X})$. For simplicity, we confine ourselves to the case of identical particles with mass m. The complete EST wave function can now be represented as

$$|\Psi_{\mathbf{P}}^{\mathrm{EST}}\rangle = U_{\mathbf{P}} |\Psi\rangle / [\langle \Psi | (2\pi)^3 \delta(\hat{\mathbf{R}}) |\Psi\rangle]^{1/2}, \tag{13}$$

where, following [18], we have introduced the projection operator ($U_{\mathbf{P}}^2 = U_{\mathbf{P}}$)

$$U_{\mathbf{P}} \equiv (2\pi)^{3/2} |\mathbf{P})(\mathbf{R} = 0|. \tag{14}$$

Given an operator $\hat{A}$, its matrix elements with the TI symmetry can be written in the form

$$\langle \Psi_{\mathbf{P}'}' | \hat{A} | \Psi_{\mathbf{P}} \rangle = \frac{\langle \Psi' | U_{\mathbf{P}'}^\dagger \hat{A} U_{\mathbf{P}} |\Psi\rangle}{[\langle \Psi' | (2\pi)^3 \delta(\hat{\mathbf{R}}) |\Psi'\rangle \langle \Psi | (2\pi)^3 \delta(\hat{\mathbf{R}}) |\Psi\rangle]^{1/2}}. \tag{15}$$

Its expectation value in the g.s.

$$|0\rangle = \int \mathrm{d}^3 P c(\mathbf{P}) U_{\mathbf{P}} |\Psi_0\rangle / [\langle \Psi_0 | (2\pi)^3 \delta(\hat{\mathbf{R}}) |\Psi_0\rangle]^{1/2}$$

is expressed in terms of the expectation value of the operator $U_{\mathbf{P}'}^\dagger \hat{A} U_{\mathbf{P}}$,

$$\langle 0| \hat{A} |0\rangle = \int \mathrm{d}^3 P \int \mathrm{d}^3 P' c^*(\mathbf{P}') c(\mathbf{P}) \frac{\langle \Psi_0 | U_{\mathbf{P}'}^\dagger \hat{A} U_{\mathbf{P}} |\Psi_0\rangle}{\langle \Psi_0 | (2\pi)^3 \delta(\hat{\mathbf{R}}) |\Psi_0\rangle}. \tag{16}$$

In addition, if $\hat{A}$ is an intrinsic operator $\hat{A}_{\text{int}}$, acting only on the space of intrinsic variables, we find

$$A_{\text{EST}} \equiv \langle 0|\hat{A}_{\text{int}}|0\rangle = \langle \Psi_0|\hat{A}_{\text{EST}}|\Psi_0\rangle/\langle \Psi_0|\delta(\hat{\mathbf{R}})|\Psi_0\rangle \quad (17)$$

$$\hat{A}_{\text{EST}} = |\mathbf{R}=0)\hat{A}_{\text{int}}(\mathbf{R}=0| = \delta(\hat{\mathbf{R}})\hat{A}_{\text{int}} = \hat{A}_{\text{int}}\delta(\hat{\mathbf{R}}). \quad (18)$$

When deriving eq. (17) we have employed the relation $(\mathbf{P}|\mathbf{P}') = \delta(\mathbf{P}-\mathbf{P}')$ and eq. (6).

It has been shown [22] that the calculation of expectation values of many-body operators like $\hat{A}_{\text{EST}}$ can be substantially simplified using the Cartesian representation. In this representation the coordinate (momentum) operator $\hat{\mathbf{r}}_\alpha$ ($\hat{\mathbf{p}}_\alpha$) of the α-th particle is expressed through the Cartesian creation and annihilation operators $\hat{\mathbf{a}}^\dagger$ and $\hat{\mathbf{a}}$,

$$\hat{\mathbf{r}} = \frac{r_0}{\sqrt{2}}(\hat{\mathbf{a}}^\dagger + \hat{\mathbf{a}}) \quad \hat{\mathbf{p}} = \mathrm{i}\frac{p_0}{\sqrt{2}}(\hat{\mathbf{a}}^\dagger - \hat{\mathbf{a}}) \quad r_0 p_0 = 1, \quad (19)$$

obeying the commutation relations

$$\left[\hat{a}_l^\dagger, \hat{a}_j^\dagger\right] = [\hat{a}_l, \hat{a}_j] = 0, \quad \left[\hat{a}_l, \hat{a}_j^\dagger\right] = \delta_{lj}, \quad (20)$$

which are the stepping stones in what follows. The indices $l, j = 1, 2, 3$ label the three Cartesian axes x, y, z.

As the "length" parameter r_0 one can choose the oscillator parameter of a suitable harmonic-oscillator basis in which the nuclear wave function is expanded. Its basis vectors $|n_x\, n_y\, n_z\rangle_1 \otimes \ldots \otimes |n_x\, n_y\, n_z\rangle_A$, where the quantum numbers n_x, n_y, n_z take the values $0, 1, \ldots$, are composed of the single-particle states

$$|n_x\, n_y\, n_z\rangle = [n_x!\, n_y!\, n_z!]^{-\frac{1}{2}} \left[\hat{a}_1^\dagger\right]^{n_x} \left[\hat{a}_2^\dagger\right]^{n_y} \left[\hat{a}_3^\dagger\right]^{n_z} |0\,0\,0\rangle, \quad (21)$$

which are the eigenstates of the Hamiltonian $\hat{H}_{\text{osc}} = \omega(\hat{\mathbf{a}}^\dagger \cdot \hat{\mathbf{a}} + \frac{3}{2})$,

$$\hat{H}_{\text{osc}}|n_x\, n_y\, n_z\rangle = \left(n_x + n_y + n_z + \tfrac{3}{2}\right)\omega\,|n_x\, n_y\, n_z\rangle,$$

where ω is the oscillation frequency along the three axes x, y and z. We use the system of units with $\hbar = c = 1$. The single-particle wave function in coordinate representation is written

$$\langle \mathbf{r} \mid n_x\, n_y\, n_z\rangle = \psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z),$$

where [34,35]

$$\psi_n(s) = \left[\sqrt{\pi}2^n n! r_0\right]^{-\frac{1}{2}} H_n(s/r_0)\exp(-s^2/2r_0^2)$$

and $H_n(x)$ is a Hermite polynomial. By definition, the oscillator parameter equals $r_0 = [m\omega]^{-\frac{1}{2}}$.

The general idea in subsequent manipulations is to bring a given operator into a form with normal ordering, in which the destruction operators $\hat{\mathbf{a}}$ are to the right with respect to the creation operators $\hat{\mathbf{a}}^\dagger$ (see sects. 4, 5). For this purpose, we will also make use of the operator identity

$$e^{\hat{A}+\hat{B}} = e^{\hat{A}}e^{\hat{B}}e^{-\frac{1}{2}\hat{C}} = e^{\hat{B}}e^{\hat{A}}e^{\frac{1}{2}\hat{C}}, \quad (22)$$

which is valid for arbitrary operators $\hat{A}$ and $\hat{B}$ if the operator $\hat{C} = [\hat{A}, \hat{B}]$ commutes with each of them. In particular,

$$e^{\mathbf{x}\cdot\hat{\mathbf{A}}+\mathbf{y}\cdot\hat{\mathbf{B}}} = e^{\mathbf{x}\cdot\hat{\mathbf{A}}}e^{\mathbf{y}\cdot\hat{\mathbf{B}}}e^{-\frac{1}{2}\mathbf{x}\cdot\mathbf{y}C} = e^{\mathbf{y}\cdot\hat{\mathbf{B}}}e^{\mathbf{x}\cdot\hat{\mathbf{A}}}e^{\frac{1}{2}\mathbf{x}\cdot\mathbf{y}C}, \quad (23)$$

if $[\hat{A}_l, \hat{B}_j] = C\delta_{lj}$ for $(l, j = 1, 2, 3)$ and C is a c-number.

3 The intrinsic density matrices and related quantities

In the preceding discussion it is implied that the operators of interest have been expressed in terms of the relevant coordinates, *e.g.*, intrinsic ones. It is not always straightforward how to do this. Here we refer mainly to the definitions of n-body density matrices (nDMs). For instance, it is a common practice [36–38] to write the 1DM in coordinate representation as the expectation value

$$\rho^{[1]}(\mathbf{r}, \mathbf{r}') = A\langle \Psi|\hat{\rho}^{[1]}(\mathbf{r}, \mathbf{r}')|\Psi\rangle \quad (24)$$

of the projection operator $\hat{\rho}^{[1]}(\mathbf{r}, \mathbf{r}')$,

$$\begin{aligned}
\hat{\rho}^{[1]}(\mathbf{r}, \mathbf{r}') &= |\mathbf{r}\rangle_{AA}\langle \mathbf{r}'| \\
&\equiv \int |\mathbf{r}_1 \ldots \mathbf{r}_{A-1}\mathbf{r}\rangle \mathrm{d}\mathbf{r}_1 \ldots \mathrm{d}\mathbf{r}_{A-1}\langle \mathbf{r}_1 \ldots \mathbf{r}_{A-1}\mathbf{r}'| \\
&= \exp(-\mathrm{i}\hat{\mathbf{p}}_A \cdot \mathbf{r})|\mathbf{r}_A = 0\rangle\langle \mathbf{r}_A = 0|\exp(\mathrm{i}\hat{\mathbf{p}}_A \cdot \mathbf{r}') \\
&= \exp(-\mathrm{i}\hat{\mathbf{p}}_A \cdot \mathbf{r})\delta(\hat{\mathbf{r}}_A)\exp(\mathrm{i}\hat{\mathbf{p}}_A \cdot \mathbf{r}')
\end{aligned}$$

in a given unit-normalized state Ψ. Its diagonal elements give the one-body density distribution $\rho(\mathbf{r}) = \rho^{[1]}(\mathbf{r}, \mathbf{r})$. The off-diagonal elements $\rho^{[1]}(\mathbf{r}, \mathbf{r}')$ provide a measure of the correlation between the probabilities to find a particle in the two positions $\mathbf{r}$ and $\mathbf{r}'$ while all the other particles are kept fixed. Such a definition seems to be satisfactory in the case of infinite systems, or systems bound by an external potential, *e.g.* the electrons of an atom. However, it is apparently problematic for finite self-bound systems like nuclei, where the constituent particles are localized around their CM due to their interaction. Therefore, we prefer to deal with the intrinsic particle distributions that depend only on intrinsic wave functions and Jacobi coordinates. Only such quantities are of physical meaning in the case of finite self-bound nonrelativistic systems. In the next subsections this will be demonstrated for the intrinsic 1DM and 2DM and related quantities.

3.1 The intrinsic one-body density matrix and momentum distribution

The intrinsic 1DM in coordinate space may be defined as

$$\rho_{\text{int}}^{[1]}(\mathbf{r}, \mathbf{r}') \equiv A\langle \Psi_{\text{int}}|\hat{\rho}_{\text{int}}^{[1]}(\mathbf{r}, \mathbf{r}')|\Psi_{\text{int}}\rangle \quad (25)$$

$$= A\langle \Psi_{\text{int}}|\boldsymbol{\xi}_{A-1} = \mathbf{r}\rangle\langle \boldsymbol{\xi}_{A-1} = \mathbf{r}'|\Psi_{\text{int}}\rangle \quad (26)$$

$$= A \int \mathrm{d}^3\xi_1 \ldots \mathrm{d}^3\xi_{A-2}\Psi_{\text{int}}^\dagger(\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_{A-2}, \mathbf{r})$$

$$\times \Psi_{\text{int}}(\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_{A-2}, \mathbf{r}'), \quad (27)$$

so that the normalization condition $\int \mathrm{d}^3 r \rho_{\mathrm{int}}^{[1]}(\mathbf{r}, \mathbf{r}) = A$ is satisfied. We would like to emphasize that this is not an "imposed" definition. It appears naturally when evaluating the dynamical FF [21] (or its diagonal part, if one uses the terminology adopted in Chapt. XI of the monograph [33]), which is related to the intrinsic OBMD [39]

$$\eta_{\mathrm{int}}(\mathbf{p}) \equiv A\langle \Psi_{\mathrm{int}} | \hat{\eta}_{\mathrm{int}}(\mathbf{p}) | \Psi_{\mathrm{int}}\rangle \qquad (28)$$

with

$$\hat{\eta}_{\mathrm{int}}(\mathbf{p}) = \delta(\mathbf{p} - \hat{\mathbf{p}}_A + \hat{\mathbf{P}}/A) = \delta(\mathbf{p} - \hat{\boldsymbol{\eta}}_{A-1}) \qquad (29)$$
$$= |\boldsymbol{\eta}_{A-1} = \mathbf{p}\rangle\langle\boldsymbol{\eta}_{A-1} = \mathbf{p}|. \qquad (30)$$

The OBMD is the Fourier transform of the 1DM $\rho_{\mathrm{int}}^{[1]}(\mathbf{r}, \mathbf{r}')$,

$$\eta_{\mathrm{int}}(\mathbf{p}) = (2\pi)^{-3} \int \mathrm{d}^3 r \mathrm{d}^3 r' \exp\left[i\mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')\right] \rho_{\mathrm{int}}^{[1]}(\mathbf{r}, \mathbf{r}'). \qquad (31)$$

See also ref. [40]. At the same time, the intrinsic one-body density $\rho_{\mathrm{int}}(\mathbf{r})$ is the Fourier transform of the elastic FF determined by eq. (9), or inversely,

$$F_{\mathrm{int}}(\mathbf{q}) = \frac{1}{A} \int \mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}} \rho_{\mathrm{int}}(\mathbf{r}) \mathrm{d}^3 r. \qquad (32)$$

From eq. (32) it follows that $\rho_{\mathrm{int}}(\mathbf{r}) = A\langle\Psi_{\mathrm{int}}|\hat{\rho}_{\mathrm{int}}(\mathbf{r})|\Psi_{\mathrm{int}}\rangle$, where

$$\hat{\rho}_{\mathrm{int}}(\mathbf{r}) = \delta(\mathbf{r} - \hat{\mathbf{r}}_A + \hat{\mathbf{R}}) = \delta(\mathbf{r} - \tfrac{A-1}{A}\hat{\boldsymbol{\xi}}_{A-1}). \qquad (33)$$

We notice that

$$\rho_{\mathrm{int}}(\mathbf{r}) = \left[\frac{A}{A-1}\right]^3 \rho_{\mathrm{int}}^{[1]}(\tfrac{A}{A-1}\mathbf{r}, \tfrac{A}{A-1}\mathbf{r}). \qquad (34)$$

In other words, the intrinsic 1DM does not have the property $\rho^{[1]}(\mathbf{r}) = \rho^{[1]}(\mathbf{r}, \mathbf{r})$ which can be justified for infinite systems, although it has often been exploited in approximate treatments of finite systems (cf., however, ref. [30], where an alternative definition of the 1DM for finite self-bound systems was proposed).

3.2 The intrinsic two-body density matrix and two-body momentum distribution

The formulation presented above will now be applied to the 2DM. In particular, we will focus on the TBMD, usually defined as the diagonal part of the 2DM in momentum space [11,31]. As we have already discussed, the relevant definitions require some revision in the case of finite, self-bound systems. Here we will consider the expectation value

$$\eta_{\mathrm{int}}^{[2]}(\mathbf{p}, \mathbf{k}) = A(A-1)\langle\Psi_{\mathrm{int}}|\delta(\hat{\mathbf{p}}_{A-1} - \tfrac{1}{A}\hat{\mathbf{P}} - \mathbf{p})$$
$$\times \delta(\hat{\mathbf{p}}_A - \tfrac{1}{A}\hat{\mathbf{P}} - \mathbf{k})|\Psi_{\mathrm{int}}\rangle$$
$$\equiv A(A-1)\langle\Psi_{\mathrm{int}}|\hat{\eta}_{\mathrm{int}}^{[2]}(\mathbf{p}, \mathbf{k})|\Psi_{\mathrm{int}}\rangle, \qquad (35)$$

that can be interpreted as the TBMD with respect to the intrinsic momentum variables. We can write for the operator $\hat{\eta}_{\mathrm{int}}^{[2]}(\mathbf{p}, \mathbf{k})$

$$\hat{\eta}_{\mathrm{int}}^{[2]}(\mathbf{p}, \mathbf{k}) = (2\pi)^{-6} \int \mathrm{d}^3\lambda_1 \mathrm{d}^3\lambda_2 \mathrm{e}^{-i\mathbf{p}\cdot\boldsymbol{\lambda}_1} \mathrm{e}^{-i\mathbf{k}\cdot\boldsymbol{\lambda}_2} \hat{E}_{\mathrm{int}}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2). \qquad (36)$$

The operator $\hat{E}_{\mathrm{int}}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)$ is expressed in terms of the Jacobi variables,

$$\hat{E}_{\mathrm{int}}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = \exp[i\boldsymbol{\lambda}_1 \cdot \hat{\boldsymbol{\eta}}_{A-2}] \exp[i(\boldsymbol{\lambda}_2 - \tfrac{1}{A-1}\boldsymbol{\lambda}_1) \cdot \hat{\boldsymbol{\eta}}_{A-1}], \qquad (37)$$

if the relations $\hat{\mathbf{p}}_A - \hat{\mathbf{P}}/A = \hat{\boldsymbol{\eta}}_{A-1}$ and $\hat{\mathbf{p}}_{A-1} - \hat{\mathbf{p}}_A = \hat{\boldsymbol{\eta}}_{A-2} - \frac{A}{A-1}\hat{\boldsymbol{\eta}}_{A-1}$ are used. Using the completeness of the $\boldsymbol{\xi}$-basis, we find

$$\hat{E}_{\mathrm{int}}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = \int \mathrm{d}^3 x \mathrm{d}^3 y \mathrm{d}^3 x' \mathrm{d}^3 y' \delta(\mathbf{x} + \boldsymbol{\lambda}_1 - \mathbf{x}')$$
$$\times \delta(\mathbf{y} + \boldsymbol{\lambda}_2 - \tfrac{1}{A-1}\boldsymbol{\lambda}_1 - \mathbf{y}')$$
$$\times \hat{\rho}_{\mathrm{int}}^{[2]}(\mathbf{x}, \mathbf{y}; \mathbf{x}', \mathbf{y}') \qquad (38)$$

with

$$\hat{\rho}_{\mathrm{int}}^{[2]}(\mathbf{x}, \mathbf{y}; \mathbf{x}', \mathbf{y}') = |\boldsymbol{\xi}_{A-2} = \mathbf{x}\rangle\langle\boldsymbol{\xi}_{A-2} = \mathbf{x}'|$$
$$\otimes |\boldsymbol{\xi}_{A-1} = \mathbf{y}\rangle\langle\boldsymbol{\xi}_{A-1} = \mathbf{y}'|.$$

The latter is the intrinsic 2DM operator in coordinate space. It follows from eq. (38) that

$$\hat{\eta}_{\mathrm{int}}^{[2]}(\mathbf{p}, \mathbf{k}) = (2\pi)^{-6} \int \mathrm{d}^3 x \mathrm{d}^3 y \mathrm{d}^3 x' \mathrm{d}^3 y' \hat{\rho}_{\mathrm{int}}^{[2]}(\mathbf{x}, \mathbf{y}; \mathbf{x}', \mathbf{y}')$$
$$\times \exp[i(\mathbf{p} + \tfrac{1}{A-1}\mathbf{k}) \cdot (\mathbf{x} - \mathbf{x}')]$$
$$\times \exp[i\mathbf{k} \cdot (\mathbf{y} - \mathbf{y}')]. \qquad (39)$$

Unlike the usual relationship

$$\hat{\eta}^{[2]}(\mathbf{p}, \mathbf{k}) \equiv \hat{\eta}^{[2]}(\mathbf{p}, \mathbf{k}; \mathbf{p}, \mathbf{k})$$
$$= (2\pi)^{-6} \int \mathrm{d}^3 r \mathrm{d}^3 s \mathrm{d}^3 r' \mathrm{d}^3 s' \mathrm{e}^{i\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}')}\mathrm{e}^{i\mathbf{k}\cdot(\mathbf{s}-\mathbf{s}')}$$
$$\times \hat{\rho}^{[2]}(\mathbf{r}, \mathbf{s}; \mathbf{r}', \mathbf{s}'),$$

where $\hat{\eta}^{[2]}(\mathbf{p}, \mathbf{k})$ is the TBMD operator and $\hat{\rho}^{[2]}(\mathbf{r}, \mathbf{s}; \mathbf{r}', \mathbf{s}')$ $(\hat{\eta}^{[2]}(\mathbf{p}, \mathbf{k}; \mathbf{p}', \mathbf{k}'))$ the 2DM operator in coordinate (momentum) space as defined, for example, in refs. [38,11], the r.h.s. of eq. (39) contains a shift $\mathbf{k}/(A-1)$ of the argument $\mathbf{p}$, which may be negligibly small when the particle number A increases. However, this is not the case for few-body systems.

4 The intrinsic density matrices and related quantities in the Cartesian representation

The intrinsic quantities are defined above in terms of operators, which can be written as products of A operators acting on the subspaces of the separate A particles. Here we will show their evaluation in the Cartesian representation.

As an illustration, let us start from the operator related to the elastic FF, eq. (9),

$$\hat{F}_{\text{int}}(\mathbf{q}) = \exp[\mathrm{i}\mathbf{q} \cdot (\hat{\mathbf{r}}_A - \hat{\mathbf{R}})] = \mathrm{e}^{-\mathrm{i}\frac{\hat{\mathbf{r}}_1}{A} \cdot \mathbf{q}} \dots \mathrm{e}^{-\mathrm{i}\frac{\hat{\mathbf{r}}_{A-1}}{A} \cdot \mathbf{q}} \mathrm{e}^{\mathrm{i}\frac{A-1}{A}\hat{\mathbf{r}}_A \cdot \mathbf{q}}.$$

Notice that

$$\hat{\rho}_{\text{int}}(\mathbf{r}) = \delta(\hat{\mathbf{r}}_A - \hat{\mathbf{R}} - \mathbf{r}) = (2\pi)^{-3} \int \mathrm{e}^{-\mathrm{i}\mathbf{q} \cdot \mathbf{r}} \hat{F}_{\text{int}}(\mathbf{q}) \mathrm{d}^3 q.$$

Using eqs. (19), (20) and (23), we find

$$\mathrm{e}^{-\mathrm{i}\frac{\hat{\mathbf{r}}}{A} \cdot \mathbf{q}} = \mathrm{e}^{-\mathrm{i}\frac{r_0}{\sqrt{2}A}\mathbf{q} \cdot (\hat{\mathbf{a}}^\dagger + \hat{\mathbf{a}})} = \mathrm{e}^{-\frac{r_0^2 q^2}{4A^2}} \mathrm{e}^{-\mathrm{i}\frac{r_0}{\sqrt{2}A}\mathbf{q} \cdot \hat{\mathbf{a}}^\dagger} \mathrm{e}^{-\mathrm{i}\frac{r_0}{\sqrt{2}A}\mathbf{q} \cdot \hat{\mathbf{a}}},$$

$$\mathrm{e}^{\mathrm{i}\frac{A-1}{A}\mathbf{r} \cdot \mathbf{q}} = \mathrm{e}^{-\frac{(A-1)^2}{A^2}\frac{r_0^2 q^2}{4}} \mathrm{e}^{\mathrm{i}\frac{A-1}{A}\frac{r_0}{\sqrt{2}}\mathbf{q} \cdot \hat{\mathbf{a}}^\dagger} \mathrm{e}^{\mathrm{i}\frac{A-1}{A}\frac{r_0}{\sqrt{2}}\mathbf{q} \cdot \hat{\mathbf{a}}},$$

whence

$$\hat{F}_{\text{int}}(\mathbf{q}) = \mathrm{e}^{-(1-\frac{1}{A})\frac{r_0^2 q^2}{4}} \hat{O}_1(\boldsymbol{\alpha}) \dots \hat{O}_{A-1}(\boldsymbol{\alpha})\hat{O}_A(\boldsymbol{\beta}),$$

where

$$\boldsymbol{\alpha} = -\mathrm{i}\frac{r_0}{\sqrt{2}A}\mathbf{q}, \qquad \boldsymbol{\beta} = \mathrm{i}\frac{r_0}{\sqrt{2}}(1 - A^{-1})\mathbf{q}. \qquad (40)$$

Henceforth we use the notation

$$\hat{O}_i(\mathbf{z}) \equiv \mathrm{e}^{-\mathbf{z}^* \cdot \hat{\mathbf{a}}_i^\dagger} \mathrm{e}^{\mathbf{z} \cdot \hat{\mathbf{a}}_i}, \qquad (41)$$

with $i = 1, 2, \dots, A$ and any complex vector $\mathbf{z}$, for a recurring operator structure. In fact, one finds that all the intrinsic one- and two-particle operators of interest contain this operator structure:

$$\hat{O}_1(\mathbf{z}) \cdots \hat{O}_{A-2}(\mathbf{z})\hat{O}_{A-1}(\mathbf{x}_2)\hat{O}_A(\mathbf{x}_1),$$

where the vectors $\mathbf{z}$, $\mathbf{x}_2$, $\mathbf{x}_1$ are related by the equation $(A-2)\mathbf{z} + \mathbf{x}_2 + \mathbf{x}_1 = 0$. In the case of one-particle operators, $\mathbf{x}_2 = \mathbf{z}$.

Therefore, according to eq. (9), the elastic FF can be written as

$$F_{\text{int}}(\mathbf{q}) = \mathrm{e}^{-(1-\frac{1}{A})\frac{r_0^2 q^2}{4}} \langle \Psi_{\text{int}} | \hat{O}_1(\boldsymbol{\alpha}) \dots \hat{O}_{A-1}(\boldsymbol{\alpha})\hat{O}_A(\boldsymbol{\beta}) | \Psi_{\text{int}} \rangle, \qquad (42)$$

where the vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are determined by eqs. (40). In the r.h.s. of the equation we find the Tassie-Barker (TB) factor $\exp(r_0^2 q^2/4A)$ [15] with the "length" parameter r_0. In our approach this factor results from the specific structure of the intrinsic operator $\hat{F}_{\text{int}}(\mathbf{q})$, being independent upon the choice of the nuclear g.s. wave function (in general, the wave function of the finite system under study). As is well known, the TB factor appears directly in calculations, where the nuclear ground state is described by the simple harmonic-oscillator model.

In the fixed-CM approximation, as we demonstrated in sect. 2 by means of eqs. (17), (18), one has to evaluate the ratio

$$F_{\text{EST}}(\mathbf{q}) = \frac{\langle \Psi_0 | \hat{F}_{\text{EST}}(\mathbf{q}) | \Psi_0 \rangle}{\langle \Psi_0 | \hat{F}_{\text{EST}}(0) | \Psi_0 \rangle} \qquad (43)$$

with

$$\hat{F}_{\text{EST}}(\mathbf{q}) = \delta(\hat{\mathbf{R}})\hat{F}_{\text{int}}(\mathbf{q}).$$

Using the integral representation $(2\pi)^3 \delta(\hat{\mathbf{R}}) = \int \exp(\mathrm{i}\boldsymbol{\xi} \cdot \hat{\mathbf{R}})\mathrm{d}^3\xi$ and applying the same technique as before, one can show that

$$\hat{F}_{\text{EST}}(\mathbf{q}) = \mathrm{e}^{-(1-\frac{1}{A})\frac{r_0^2 q^2}{4}} \int \mathrm{d}^3\xi \mathrm{e}^{-r_0^2\xi^2/4A}$$
$$\times \hat{O}_1(\boldsymbol{\alpha}') \dots \hat{O}_{A-1}(\boldsymbol{\alpha}')\hat{O}_A(\boldsymbol{\beta}'), \qquad (44)$$

where

$$\boldsymbol{\alpha}' = \mathrm{i}\frac{r_0}{\sqrt{2}A}(\boldsymbol{\xi} - \mathbf{q}), \qquad \boldsymbol{\beta}' = \mathrm{i}\frac{r_0}{\sqrt{2}}\left[\frac{\boldsymbol{\xi}}{A} + (1 - A^{-1})\mathbf{q}\right].$$

When calculating expectation values like those in eq. (43), the representation (44) is especially helpful if the wave function Ψ_0 is a Slater determinant or a linear combination of Slater determinants. This has been demonstrated in particular in refs. [22,41], where the single-particle orbitals entering the Slater determinant are eigenfunctions of a harmonic-oscillator potential with oscillator parameter r_0. Recently, similar calculations have been carried out beyond the HOM [29] with the single-particle orbitals approximated by a truncated expansion in the Cartesian basis vectors of eq. (21).

The intrinsic 1DM operator can also be expressed in terms of the Cartesian operators $\hat{\mathbf{a}}^\dagger$ and $\hat{\mathbf{a}}$. We can rewrite the operator $\hat{\rho}_{\text{int}}^{[1]}(\mathbf{r}, \mathbf{r}')$ from eq. (25) as

$$\hat{\rho}_{\text{int}}^{[1]}(\mathbf{r}, \mathbf{r}') = \mathrm{e}^{-\mathrm{i}\hat{\boldsymbol{\eta}}_{A-1} \cdot \mathbf{r}} \delta(\hat{\boldsymbol{\xi}}_{A-1}) \mathrm{e}^{\mathrm{i}\hat{\boldsymbol{\eta}}_{A-1} \cdot \mathbf{r}'}$$
$$= (2\pi)^{-3} \int \mathrm{d}^3\lambda \exp[-\mathrm{i}(\hat{\mathbf{p}}_A - \frac{\hat{\mathbf{P}}}{A}) \cdot \mathbf{r}]$$
$$\times \exp[\mathrm{i}\frac{A-1}{A}\boldsymbol{\lambda} \cdot (\hat{\mathbf{r}}_A - \hat{\mathbf{R}})]$$
$$\times \exp[\mathrm{i}(\hat{\mathbf{p}}_A - \frac{\hat{\mathbf{P}}}{A}) \cdot \mathbf{r}']. \qquad (45)$$

As before, the general idea is to bring this operator in a form with normal ordering, where the destruction operators $\hat{\mathbf{a}}_A$ are to the right with respect to the creation operators $\hat{\mathbf{a}}_A^\dagger$. To do this, we note that

$$\exp\left[\mathrm{i}\frac{A}{A-1}\boldsymbol{\lambda} \cdot (\hat{\mathbf{r}}_A - \hat{\mathbf{R}})\right] = \exp[-\frac{A}{A-1}\frac{r_0^2\lambda^2}{4}]$$
$$\times \exp[\mathrm{i}\frac{A}{A-1}\frac{r_0}{\sqrt{2}A}\boldsymbol{\lambda} \cdot (\hat{\mathbf{a}}_A^\dagger - \frac{\hat{\mathbf{D}}^\dagger}{A})]$$
$$\times \exp[\mathrm{i}\frac{A}{A-1}\frac{r_0}{\sqrt{2}A}\boldsymbol{\lambda} \cdot (\hat{\mathbf{a}}_A - \frac{\hat{\mathbf{D}}}{A})],$$

where $\hat{\mathbf{D}} = \sum_{\alpha=1}^A \hat{\mathbf{a}}_\alpha$ is the "collective" destruction operator with the property $[\hat{D}_l, \hat{D}_j^\dagger] = A\delta_{lj}$ $(l, j = 1, 2, 3)$. After some modest effort we arrive at the following result:

$$\hat{\rho}_{\text{int}}^{[1]}(\mathbf{r}, \mathbf{r}') = (2\pi)^{-3}\exp[-\frac{A-1}{A}\frac{p_0^2}{4}(\mathbf{r} - \mathbf{r}')^2]$$
$$\times \int \mathrm{d}^3\lambda \exp[-\frac{A-1}{A}\frac{r_0^2}{4}\lambda^2] \exp[-\mathrm{i}\frac{\boldsymbol{\lambda}}{2} \cdot (\mathbf{r} + \mathbf{r}')]$$
$$\times \exp[(\frac{p_0}{\sqrt{2}}(\mathbf{r} - \mathbf{r}') + \mathrm{i}\frac{A}{A-1}\frac{r_0}{\sqrt{2}A}\boldsymbol{\lambda}) \cdot (\hat{\mathbf{a}}^\dagger - \frac{\hat{\mathbf{D}}^\dagger}{A})]$$
$$\times \exp[(-\frac{p_0}{\sqrt{2}}(\mathbf{r} - \mathbf{r}') + \mathrm{i}\frac{A}{A-1}\frac{r_0}{\sqrt{2}A}\boldsymbol{\lambda}) \cdot (\hat{\mathbf{a}} - \frac{\hat{\mathbf{D}}}{A})].$$

The operator $\hat{\rho}_{\text{EST}}^{[1]}(\mathbf{r}, \mathbf{r}') = \delta(\hat{\mathbf{R}})\hat{\rho}_{\text{int}}^{[1]}(\mathbf{r}, \mathbf{r}')$ can be represented in a similar way. Furthermore, taking into account

eq. (31), the corresponding OBMD operator $\hat{\eta}_{\text{int}}(\mathbf{p})$ equals

$$
\begin{aligned}
\hat{\eta}_{\text{int}}(\mathbf{p}) = (2\pi)^{-3} \int \mathrm{d}^3 y \exp[i\mathbf{p} \cdot \mathbf{y}] \exp[-\tfrac{A-1}{A}\tfrac{p_0^2}{4}y^2] \\
\times \exp[\tfrac{p_0}{\sqrt{2}}\mathbf{y} \cdot (\hat{\mathbf{a}}^\dagger - \tfrac{\hat{\mathbf{D}}^\dagger}{A})] \\
\times \exp[\tfrac{p_0}{\sqrt{2}}\mathbf{y} \cdot (\hat{\mathbf{a}} - \tfrac{\hat{\mathbf{D}}}{A})] \quad (46)
\end{aligned}
$$

$$
\begin{aligned}
= (2\pi)^{-3} \int \mathrm{d}^3 y \exp[i\mathbf{p} \cdot \mathbf{y}] \exp[-\tfrac{A-1}{A}\tfrac{p_0^2}{4}y^2] \\
\times \hat{O}_1(\boldsymbol{\chi}) \ldots \hat{O}_{A-1}(\boldsymbol{\chi}) \hat{O}_A(\boldsymbol{\gamma}). \quad (47)
\end{aligned}
$$

As for the operator $\hat{\eta}_{\text{EST}}(\mathbf{p})$, we find

$$
\begin{aligned}
\hat{\eta}_{\text{EST}}(\mathbf{p}) = (2\pi)^{-3} \int \mathrm{d}^3 y \mathrm{d}^3 \xi \exp[-\tfrac{A-1}{A}\tfrac{p_0^2}{4}y^2] \\
\times \exp[-\tfrac{1}{A}\tfrac{r_0^2}{4}\xi^2] \exp[i\mathbf{y} \cdot \mathbf{p}] \\
\times \hat{O}_1(\boldsymbol{\chi}') \ldots \hat{O}_{A-1}(\boldsymbol{\chi}') \hat{O}_A(\boldsymbol{\gamma}'), \quad (48)
\end{aligned}
$$

where

$$
\boldsymbol{\gamma} = -\tfrac{p_0}{\sqrt{2}}(1 - \tfrac{1}{A})\mathbf{y}, \qquad \boldsymbol{\chi} = \tfrac{p_0}{\sqrt{2A}}\mathbf{y} \quad (49)
$$

$$
\boldsymbol{\gamma}' = \mathrm{i}\tfrac{r_0}{\sqrt{2}A}\boldsymbol{\xi} - \tfrac{p_0}{\sqrt{2}}(1 - \tfrac{1}{A})\mathbf{y}, \quad \boldsymbol{\chi}' = \mathrm{i}\tfrac{r_0}{\sqrt{2}A}\boldsymbol{\xi} + \tfrac{p_0}{\sqrt{2A}}\mathbf{y}. \quad (50)
$$

With the help of eqs. (17), (18), (48), (50), the OBMD

$$
\eta_{\text{EST}}(\mathbf{p}) = A \frac{\langle \Psi_0 | \hat{\eta}_{\text{EST}}(\mathbf{p}) | \Psi_0 \rangle}{\langle \Psi_0 | \delta(\hat{\mathbf{R}}) | \Psi_0 \rangle}
$$

can be evaluated like the elastic FF $F_{\text{EST}}(q)$ in the fixed-CM approximation.

By comparing the expressions

$$
\hat{\eta}_{\text{int}}(\mathbf{p}) = (2\pi)^{-3} \int \mathrm{d}\mathbf{y} \exp[i\mathbf{y} \cdot \mathbf{p}] \exp\left[i\mathbf{y} \cdot \left(\hat{\mathbf{p}}_A - \tfrac{1}{A}\hat{\mathbf{P}} \right) \right],
$$

following from eq. (29), and

$$
\hat{F}_{\text{int}}(\mathbf{y}) = \exp[i\mathbf{y} \cdot (\hat{\mathbf{r}}_A - \hat{\mathbf{R}})],
$$

we realize that it is sufficient to apply our algebraic technique to the operator

$$
\exp\left[i\mathbf{y} \cdot \left(b_1 \left(\hat{\mathbf{a}}_A^\dagger - \tfrac{1}{A}\hat{\mathbf{D}}^\dagger \right) + b_2 \left(\hat{\mathbf{a}}_A - \tfrac{1}{A}\hat{\mathbf{D}} \right) \right) \right].
$$

Then we can generate the intrinsic operators $\hat{F}_{\text{int}}$ (or $\hat{\rho}_{\text{int}}$) and $\hat{\eta}_{\text{int}}$ by changing the values of b_1, b_2.

Similar manipulations lead to the following expression for the operator $\hat{E}_{\text{int}}$ that enters in the definition of the TBMD operator $\hat{\eta}_{\text{int}}^{[2]}(\mathbf{p}, \mathbf{k})$ (see eqs. (36)-(38)):

$$
\begin{aligned}
\hat{E}_{\text{int}}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = \mathrm{e}^{-\frac{p_0^2 \lambda^2}{8}} \mathrm{e}^{-\frac{A-2}{A}\frac{p_0^2 \Lambda^2}{2}} \\
\times \hat{O}_1(\boldsymbol{\zeta}) \ldots \hat{O}_{A-2}(\boldsymbol{\zeta}) \hat{O}_{A-1}(\boldsymbol{\gamma}_2) \hat{O}_A(\boldsymbol{\gamma}_1), \quad (51)
\end{aligned}
$$

where

$$
\boldsymbol{\gamma}_1 = \tfrac{p_0}{\sqrt{2}}(\tfrac{A-2}{A}\boldsymbol{\Lambda} - \tfrac{1}{2}\boldsymbol{\lambda}), \qquad \boldsymbol{\gamma}_2 = \tfrac{p_0}{\sqrt{2}}(\tfrac{A-2}{A}\boldsymbol{\Lambda} + \tfrac{1}{2}\boldsymbol{\lambda}),
$$

$$
\boldsymbol{\zeta} = -\sqrt{2}\tfrac{p_0}{A}\boldsymbol{\Lambda}.
$$

We have set $\boldsymbol{\Lambda} = (\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2)/2$ and $\boldsymbol{\lambda} = \boldsymbol{\lambda}_1 - \boldsymbol{\lambda}_2$. In order to obtain the TBMD operator in the fixed-CM approximation, one first needs to evaluate

$$
\hat{E}_{\text{EST}}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = (2\pi)^3 \delta(\hat{\mathbf{R}}) \hat{E}_{\text{int}}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2). \quad (52)
$$

Again, after some algebra one can show that

$$
\begin{aligned}
\hat{E}_{\text{EST}}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = \int \mathrm{d}^3 \kappa \mathrm{e}^{-r_0^2 \kappa^2/4A} \mathrm{e}^{-p_0^2 \lambda^2/8} \mathrm{e}^{-\frac{A-2}{2A}p_0^2 \Lambda^2} \\
\times \hat{O}_1(\boldsymbol{\zeta}') \ldots \hat{O}_{A-2}(\boldsymbol{\zeta}') \hat{O}_{A-1}(\boldsymbol{\gamma}_2') \hat{O}_A(\boldsymbol{\gamma}_1'), \quad (53)
\end{aligned}
$$

where

$$
\boldsymbol{\gamma}_1' = \mathrm{i}\tfrac{r_0}{\sqrt{2}A}\boldsymbol{\kappa} + \tfrac{p_0}{\sqrt{2}}(\tfrac{A-2}{A}\boldsymbol{\Lambda} - \tfrac{1}{2}\boldsymbol{\lambda}),
$$

$$
\boldsymbol{\gamma}_2' = \mathrm{i}\tfrac{r_0}{\sqrt{2}A}\boldsymbol{\kappa} + \tfrac{p_0}{\sqrt{2}}(\tfrac{A-2}{A}\boldsymbol{\Lambda} + \tfrac{1}{2}\boldsymbol{\lambda})
$$

and

$$
\boldsymbol{\zeta}' = \mathrm{i}\tfrac{r_0}{\sqrt{2}A}\boldsymbol{\kappa} - \sqrt{2}\tfrac{p_0}{A}\boldsymbol{\Lambda}.
$$

The respective TBMD can be written as

$$
\begin{aligned}
\eta_{\text{EST}}^{[2]}(\mathbf{p}, \mathbf{k}) = (2\pi)^{-6} A(A-1) \int \mathrm{d}^3 \Lambda \mathrm{d}^3 \lambda \mathrm{e}^{-\mathrm{i}\mathbf{p} \cdot (\boldsymbol{\Lambda} + \boldsymbol{\lambda}/2)} \\
\times \mathrm{e}^{-\mathrm{i}\mathbf{k} \cdot (\boldsymbol{\Lambda} - \boldsymbol{\lambda}/2)} \frac{N(\boldsymbol{\Lambda} + \boldsymbol{\lambda}/2, \boldsymbol{\Lambda} - \boldsymbol{\lambda}/2)}{N(0, 0)} \quad (54)
\end{aligned}
$$

with

$$
N(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = \langle \Psi_0 | \hat{E}_{\text{EST}}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) | \Psi_0 \rangle.
$$

One can verify that this distribution meets the sequential relation

$$
\int \eta_{\text{EST}}^{[2]}(\mathbf{p}, \mathbf{k}) \mathrm{d}^3 p = (A-1) \eta_{\text{EST}}(\mathbf{k}). \quad (55)
$$

The exposed method can be helpful in more general situations when one has to handle translationally invariant states of the kind

$$
|\Psi_{\mathbf{P}}^{\text{G}}\rangle = |\mathbf{P}) |\Psi_{\text{int}}^{\text{G}}\rangle, \quad (56)
$$

with the normalized state vector in the $(3A - 3)$-dimensional intrinsic Hilbert space $|\Psi_{\text{int}}^{\text{G}}\rangle$ defined by

$$
|\Psi_{\text{int}}^{\text{G}}\rangle = \frac{(\text{G}|\Psi_0\rangle}{[\langle \Psi_0 | \text{G})(\text{G}|\Psi_0\rangle]^{1/2}}. \quad (57)
$$

Here, $|\text{G})$ is any arbitrary vector in the CM space so that the scalar product $(\text{G}|\Psi_0\rangle$ represents integration of the CM variable only. Such general cases were considered in ref. [18]. The expectation value of the intrinsic operator $\hat{A}_{\text{int}}$ in the state $|0_{\text{G}}\rangle = \int \mathrm{d}^3 P c(\mathbf{P})|\mathbf{P}) |\Psi_{\text{int}}^{\text{G}}\rangle$ is

$$
A_{\text{G}} \equiv \langle 0_G | \hat{A}_{\text{int}} | 0_G \rangle = \langle \Psi_0 | \hat{A}_{\text{G}} | \Psi_0 \rangle / \langle \Psi_0 | \text{G})(\text{G}|\Psi_0\rangle, \quad (58)
$$

$$
\hat{A}_{\text{G}} = |\text{G})\hat{A}_{\text{int}}(\text{G}| = |\text{G})(\text{G}|\hat{A}_{\text{int}} = \hat{A}_{\text{int}}|\text{G})(\text{G}|. \quad (59)
$$

(Cf. eqs. (24)-(25) in ref. [19].)

The operator $\hat{A}_{\text{G}}$ can be represented in the two equivalent forms

$$
\hat{A}_{\text{G}} = \int \mathrm{d}^3 P \int \mathrm{d}^3 P' G^*(\mathbf{P}') G(\mathbf{P}) \hat{M}(\mathbf{P}, \mathbf{P}') \hat{A}_{\text{int}} \quad (60)
$$

and

$$\hat{A}_G = \int \mathrm{d}^3 R \int \mathrm{d}^3 R' \tilde{G}^*(\mathbf{R}') \tilde{G}(\mathbf{R}) \hat{M}(\mathbf{R}, \mathbf{R}') \hat{A}_{\mathrm{int}} \quad (61)$$

with the projection operators $\hat{M}(\mathbf{P}, \mathbf{P}') = |\mathbf{P})(\mathbf{P}'|$ and $\hat{M}(\mathbf{R}, \mathbf{R}') = |\mathbf{R})(\mathbf{R}'|$. The functions $G(\mathbf{P}) = (\mathbf{P}|G)$ and $\tilde{G}(\mathbf{R}) = (\mathbf{R}|G)$ represent the vector $|G)$ in the two bases composed of the $\mathbf{P}$-eigenvectors (*i.e.*, $\hat{\mathbf{P}}|\mathbf{P}) = \mathbf{P}|\mathbf{P})$) and $\mathbf{R}$-eigenvectors (*i.e.*, $\hat{\mathbf{R}}|\mathbf{R}) = \mathbf{R}|\mathbf{R})$), respectively.

So far, the function G has been totally arbitrary. If we set $\tilde{G}(\mathbf{R}) = \delta(\mathbf{R})$, *i.e.*, $|G) = |\mathbf{R} = 0)$ and $G(\mathbf{P}) = (2\pi)^{-3/2}$, we repeat the EST prescription with the intrinsic state $|\Psi_{\mathrm{int}}^{\mathrm{EST}}\rangle$, eq. (12), and the expectation value:

$$A_{\mathrm{EST}} = \frac{\langle \Psi_0 | \delta(\hat{\mathbf{R}}) \hat{A}_{\mathrm{int}} | \Psi_0 \rangle}{\langle \Psi_0 | \delta(\hat{\mathbf{R}}) | \Psi_0 \rangle}. \quad (62)$$

If, on the other hand, we set $G(\mathbf{P}) = \delta(\mathbf{P})$, *i.e.*, $|G) = |\mathbf{R} = 0)$ and $\tilde{G}(\mathbf{R}) = (2\pi)^{-3/2}$, we arrive at the no-fixed-CM approximation after Peierls and Yoccoz [17] in constructing the intrinsic state

$$|\Psi_{\mathrm{int}}^{\mathrm{PY}}\rangle = \frac{(\mathbf{P} = 0|\Psi_0\rangle}{[\langle \Psi_0 | \delta(\hat{\mathbf{P}}) | \Psi_0 \rangle]^{1/2}}. \quad (63)$$

The expectation value of interest is

$$A_{\mathrm{PY}} = \frac{\langle \Psi_0 | \delta(\hat{\mathbf{P}}) \hat{A}_{\mathrm{int}} | \Psi_0 \rangle}{\langle \Psi_0 | \delta(\hat{\mathbf{P}}) | \Psi_0 \rangle}. \quad (64)$$

Note that within the PY approach the TI state vector for a nucleus moving with momentum $\mathbf{P}$ is approximated by $|\Psi_{\mathbf{P}}^{\mathrm{PY}}\rangle = |\mathbf{P})(\mathbf{P}|\Psi_0\rangle$. Such a model is incompatible with the Galileo invariance requirement. The restoration of Galileo invariance was discussed by Peierls and Thouless [42]. The unit-normalized vector given by eq. (63) corresponds to the PY prescription for the nucleus rest frame only. In this context we would like to refer again to the several calculations in nuclei with the ansatz (63) performed recently by Schmid and his colleagues [26,27].

Previous experience prompts us to unify forthcoming calculations of quantities like (62) and (64) by introducing one δ-function, $\delta(c_1 \hat{\mathbf{D}}^\dagger + c_2 \hat{\mathbf{D}})$, where c_1, c_2 c-number parameters, instead of the two functions $\delta(\hat{\mathbf{R}})$ and $\delta(\hat{\mathbf{P}})$. Along these guidelines, one is able to build up a family of generating functions which can be calculated with the help of the algebraic technique developed in this article.

Finally, the following representation:

$$\hat{M}(\mathbf{R}, \mathbf{R}') = \exp(-\mathrm{i}\hat{\mathbf{P}} \cdot \mathbf{R}) \delta(\hat{\mathbf{R}}) \exp(\mathrm{i}\hat{\mathbf{P}} \cdot \mathbf{R}')$$

might be useful in calculations employing a refined determination of the weight function G. In particular, as outlined in ref. [18], the "best" G must be chosen so as to minimize the expectation value of the intrinsic Hamiltonian. See also ref. [20], devoted to an optimal separation of CM motion in many-body systems.

Closing this section, we should note that within the simple HOM, where the single-particle states are described

as pure harmonic-oscillator wave functions, the total wave function is always a product of the CM wave function and an intrinsic one. Therefore, for intrinsic quantities, one gets the same results with or without the use of a projection technique.

5 The intrinsic one-body and two-body momentum distributions: Application to ^{4}He and discussion

The many-particle operators encountered so far have much in common with each other owing to the operator structure appearing in each of them,

$$\hat{O}_1(\mathbf{z}) \cdots \hat{O}_{A-2}(\mathbf{z}) \hat{O}_{A-1}(\mathbf{x}_2) \hat{O}_A(\mathbf{x}_1).$$

Let us derive their expectation value in the independent-particle model. The wave function is then a Slater determinant $|\mathrm{Det}\rangle$ (eq. (10)). The following formal result is straightforward:

$$\langle \mathrm{Det} | \hat{O}_1(\mathbf{z}) \cdots \hat{O}_{A-2}(\mathbf{z}) \hat{O}_{A-1}(\mathbf{x}_2) \hat{O}_A(\mathbf{x}_1) | \mathrm{Det} \rangle$$
$$= \langle \mathrm{Det}(-\mathbf{x}_1, -\mathbf{x}_2, -\mathbf{z}) | \mathrm{Det}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{z}) \rangle, \quad (65)$$

where

$$|\mathrm{Det}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{z})\rangle \equiv \frac{1}{\sqrt{A!}} \sum_{\hat{\mathcal{P}} \in S_A} \epsilon_{\mathcal{P}} \hat{\mathcal{P}} \{ \mathrm{e}^{\mathbf{z} \cdot \hat{\mathbf{a}}_1} |\phi_{p_1}(1)\rangle \cdots \mathrm{e}^{\mathbf{z} \cdot \hat{\mathbf{a}}_{A-2}}$$
$$\times |\phi_{p_{A-2}}(A-2)\rangle \mathrm{e}^{\mathbf{x}_2 \cdot \hat{\mathbf{a}}_{A-1}} |\phi_{p_{A-1}}(A-1)\rangle \mathrm{e}^{\mathbf{x}_1 \cdot \hat{\mathbf{a}}_A} |\phi_{p_A}(A)\rangle \}. \quad (66)$$

The r.h.s. of eq. (65) can be represented as the sum of terms containing the matrix elements $\langle \phi_{p_i} | \hat{O}_m | \phi_{p_j} \rangle$ $(i, j, m = 1, 2, \ldots A)$. Note that the permutations $\hat{\mathcal{P}}$ are related to the single-particle states (not the nucleon labels). In the simplest case of the $0s^4$ configuration, that we encounter in ^{4}He nucleus, all we need is to evaluate the matrix element $\langle 0s | \hat{O}_m | 0s \rangle$. We have written $|\phi_{0s}\rangle = |0s\rangle |\sigma\tau\rangle$, where $|0s\rangle$ and $|\sigma\tau\rangle$ are the space and spin-isospin parts, respectively, of $|\phi_{0s}\rangle$. In the HOM, $\phi_{0s}(\mathbf{r}) \equiv \langle \mathbf{r} | 0s \rangle = (\sqrt{\pi} b)^{-3/2} \mathrm{e}^{-r^2/2b^2}$, where $b = r_0 = 1/p_0$ is the harmonic-oscillator parameter. The state $|0s\rangle$ coincides with the lowest-energy state $|000\rangle$ which is the "vacuum" of the Cartesian representation, *viz*, $\hat{\mathbf{a}}|000\rangle = 0$. As a consequence, the radial matrix element of interest is equal to unity and

$$\langle 0s^4 | \hat{O}_1(\mathbf{z}) \hat{O}_2(\mathbf{z}) \hat{O}_3(\mathbf{x}_2) \hat{O}_4(\mathbf{x}_1) | 0s^4 \rangle = 1, \quad (67)$$

rendering the rest of the calculation trivial. We are now in a position to write down expressions for all the quantities considered, in the case of ^{4}He. In particular, performing the necessary integrations and taking into account the normalization of each quantity, we get for the intrinsic OBMD and TBMD in the EST prescription

$$\eta_{\mathrm{EST}}(\mathbf{p}) = \eta_{\mathrm{EST}}(p) = \frac{4^{3/2} 4 b^3}{3^{3/2} \pi^{3/2}} \mathrm{e}^{-\frac{4}{3} p^2 b^2}, \quad (68)$$

$$\eta_{\mathrm{EST}}^{[2]}(\mathbf{p}, \mathbf{k}) = \frac{2^{3/2} 12 b^6}{\pi^3} \mathrm{e}^{-\frac{3}{2} p^2 b^2} \mathrm{e}^{-\frac{3}{2} k^2 b^2} \mathrm{e}^{-\mathbf{p} \cdot \mathbf{k} b^2}. \quad (69)$$

Regarding the properties of these distributions, we find that they satisfy the relation (55):

$$\int \eta_{\mathrm{EST}}^{[2]}(\mathbf{p}, \mathbf{k}) \mathrm{d}^3 p = 3\eta_{\mathrm{EST}}(\mathbf{k}).$$

In the simple HOM (with the CM correction not included) we have [11,31,43]

$$\eta(\mathbf{p}) = \eta(p) = \frac{4b^3}{\pi^{3/2}} \mathrm{e}^{-p^2 b^2}, \tag{70}$$

$$\eta^{[2]}(\mathbf{p}, \mathbf{k}) = \frac{12b^6}{\pi^3} \mathrm{e}^{-p^2 b^2} \mathrm{e}^{-k^2 b^2}. \tag{71}$$

We notice that, within the HOM, the expression for the OBMD in the EST approach, eq. (68), is the same as the one without the CM fixation, eq. (70), if one substitutes b with $\sqrt{4/3}b$. This results in a narrowing of distribution $\eta_{\mathrm{EST}}(\mathbf{p})/\eta_{\mathrm{EST}}(0)$ compared to $\eta(\mathbf{p})/\eta(0)$. But this observation does not mean that the CM correction reduces to the renormalization $b \to b_{CM} = \sqrt{4/3}b$. In this connection, let us recall the work [22], where the charge FF and the dynamic FF of ^{4}He were calculated under some simplifying assumptions using the same oscillatory wave function and the EST prescription. Its main discovery was the narrowing of both the intrinsic density and OBMD due to the CM correction. In order to demonstrate this phenomenon, let us write the nucleon density in the EST approach,

$$\rho_{\mathrm{EST}}(r) = 4\frac{\exp(-\frac{r^2}{\bar{r}_0^2})}{\pi^{3/2}\bar{r}_0^3}, \tag{72}$$

that follows directly from previous derivations and rewrite eq. (70) in the form

$$\eta_{\mathrm{EST}}(p) = 4\frac{\exp(-\frac{p^2}{\bar{p}_0^2})}{\pi^{3/2}\bar{p}_0^3}. \tag{73}$$

Here we have come back to the oscillator parameters r_0 and p_0 and introduced the new values $\bar{r}_0 = \sqrt{\frac{A-1}{A}}r_0$ and $\bar{p}_0 = \sqrt{\frac{A-1}{A}}p_0$. Meanwhile, we write $\sqrt{\frac{A-1}{A}}$ instead of $\sqrt{3/4}$ to point out a trend in the A-dependence of this effect of center-of-mass motion. We notice that the corresponding quantities in the simple HOM

$$\rho(r) = 4\frac{\exp(-\frac{r^2}{r_0^2})}{\pi^{3/2}r_0^3},$$

$$\eta(p) = 4\frac{\exp(-\frac{p^2}{p_0^2})}{\pi^{3/2}p_0^3}$$

are obtained from eqs. (72)-(73) when $A \to \infty$.

Thus we see (cf. [22]) that the inclusion of CM corrections gives rise to the two renormalizations, $r_0 \to \bar{r}_0 = \sqrt{\frac{A-1}{A}}r_0$ and $p_0 \to \bar{p}_0 = \sqrt{\frac{A-1}{A}}p_0$, of the oscillator parameter values, r_0 and p_0, in the density and momentum distributions calculated within the simple HOM. Evidently,

such changes are not accounted for by a hasty replacement of b by $\sqrt{4/3}b$. (Therefore, the discussion in ref. [44], p. 263, on the results of ref. [22] is incomplete.) One may say that we encounter a specific effect of shrinking of the density distribution $\rho(r)$ and the momentum distribution $\eta(p)$. The term "shrinking" implies that each of these densities, after being CM corrected, increases in its central but decreases in its peripheral region. As has been shown in the past [22,45], such a simultaneous change of the one-body distributions plays an essential role in getting a fair treatment of the data on elastic and inelastic electron scattering off ^{4}He. Notice that the product $\bar{r}_0\bar{p}_0 = 1-1/A \neq 1$, unlike the relation $r_0 p_0 = 1$. The commutation rules for the intrinsic coordinates $\mathbf{r}' = \mathbf{r} - \mathbf{R}$ and conjugate momenta $\mathbf{p}' = \mathbf{p} - \mathbf{P}/A$

$$[(\hat{\mathbf{r}}')_i, (\hat{\mathbf{p}}')_j] = \mathrm{i}\delta_{ij}(1 - 1/A), \quad i, j = 1, 2, 3$$

and the corresponding uncertainty principle are related to this deviation from unity [41]. Thus, the uncertainty principle is not contradicted by the simultaneous shrinking of the density and momentum distribution (see also [46], Lect. I, Suppl. C).

A similar shrinking is encountered also in the two-dimensional surface given by the function $\eta_{\gamma\mathrm{EST}}^{[2]}(p, k) \equiv \eta_{\mathrm{EST}}^{[2]}(\mathbf{p}, \mathbf{k})$ *vs.* the one given by $\eta_{\gamma}^{[2]}(p, k) \equiv \eta^{[2]}(\mathbf{p}, \mathbf{k})$, at each value of the correlation angle γ. This effect is clearly visible if the TBMD of ^{4}He is represented as

$$\eta_{\gamma\mathrm{EST}}^{[2]}(p, k) = \left[\frac{A}{A-2}\right]^{3/2} \eta_{\gamma}^{[2]}(p, k)\mathrm{e}^{-\frac{1}{A-2}\frac{(\mathbf{p}+\mathbf{k})^2}{p_0^2}}.$$

Now, in order to compare numerically calculations based upon the formulae (69) and (71), we adopt, as in ref. [22], the following way of determining the harmonic-oscillator parameter b. We start with a general expression for the charge FF $F_C(q) = f_p(q)F(q)$ of a nucleus, where $f_p(q)$ the correction factor for the finite proton size, corresponding to the proton mean-square radius $b_p \approx 0.8\,\mathrm{fm}$, as in ref. [43]. The FF of the ^{4}He nucleus equals $F(q) = \exp(-r_0^2 q^2/4)$ in the simple HOM and $F_{\mathrm{EST}}(q) = \exp(-\bar{r}_0^2 q^2/4)$ in the HOM with CM corrections. In each of the two models the value of the adjustable parameter $r_0 = b$ will be determined by the requirement to reproduce the experimental value of the charge mean-square radius of ^{4}He, $r_{\mathrm{rms}} = 1.67\,\mathrm{fm}$ [47]. Then in the simple HOM ($b = b_0$) we have

$$r_{\mathrm{rms}}^2 = \frac{3}{2}b_0^2 + b_p^2$$

and therefore $b_0 = 1.197\,\mathrm{fm}$. When CM motion corrections are taken into account ($b = b_{\mathrm{CM}}$),

$$r_{\mathrm{rms}}^2 = \frac{3}{2}\frac{A-1}{A}b_{\mathrm{CM}}^2 + b_p^2$$

and therefore $b_{\mathrm{CM}} = \frac{A}{A-1}b_0 = 1.382\,\mathrm{fm}$. By adopting this fitting we therefore get the identical q-dependence, $F_{\mathrm{EST}}(\mathbf{q}) = F(\mathbf{q}) = \exp(-b_0^2 q^2/4)$, while the difference
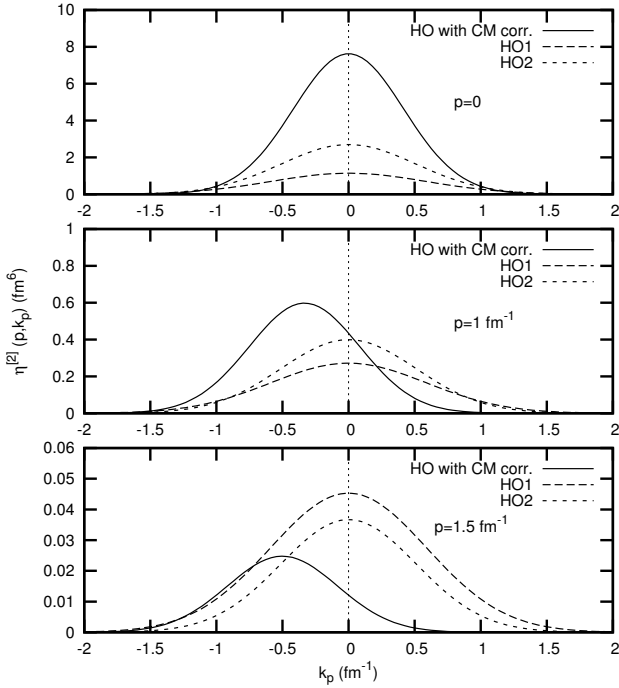
Fig. 1. The TBMD of ^{4}He for $\mathbf{p} \parallel \mathbf{k}$ and $p = 0, 1, 1.5\,\mathrm{fm}^{-1}$, as a function of k_p, where $\mathbf{k} = k_p\hat{p}$, in the HOM model. Full line: with CM motion effects taken into account within the EST approach, eq. (69), with $b = b_{CM}$; long-dashed line: in the simple HOM, eq. (71), with $b = b_0$ (HO1); short-dashed line: in the simple HOM, eq. (71), with $b = b_{CM}$ (HO2), from refs. [11,31].

between the respective OBMDs becomes more considerable. In fact, we have

$$\eta_{\mathrm{EST}}(p) = 4 \left[\frac{A}{A-1} \right]^3 \frac{b_0{}^3}{\pi^{3/2}} \exp\left(- \left[\frac{A}{A-1} \right]^2 p^2 b_0{}^2 \right)$$

vs.

$$\eta(p) = 4 \frac{b_0{}^3}{\pi^{3/2}} \exp(-p^2 b_0{}^2).$$

The former differs from the latter by a substitution $b_0 \to \frac{A}{A-1} b_0$.

In fig. 1 the TBMD of ^{4}He is shown for $p = 0, 1, 1.5\,\mathrm{fm}^{-1}$ as a function of k_p, where $\mathbf{k}$ is parallel to $\mathbf{p}$ and $\mathbf{k} = k_p\hat{p}$, *i.e.*, k_p is positive (negative) for $\mathbf{k}$ in the same (opposite) direction as $\mathbf{p}$. The TBMD that we have calculated using the EST method, *i.e.*, with CM motion corrections ($b = b_{CM}$), is plotted with full lines, while the TBMD within the simple HOM with $b = b_0$ is plotted with long-dashed lines ("HO1") and the TBMD within the simple HOM with $b = b_{CM}$ is plotted with short-dashed lines ("HO2") taken from refs. [11,31]. One can observe the shift of the peak from $k_p = 0$ towards negative k_p's, for $p \neq 0$, due to the correlation induced by the fixed center of mass.

In refs. [11,31] the dimensionless quantity

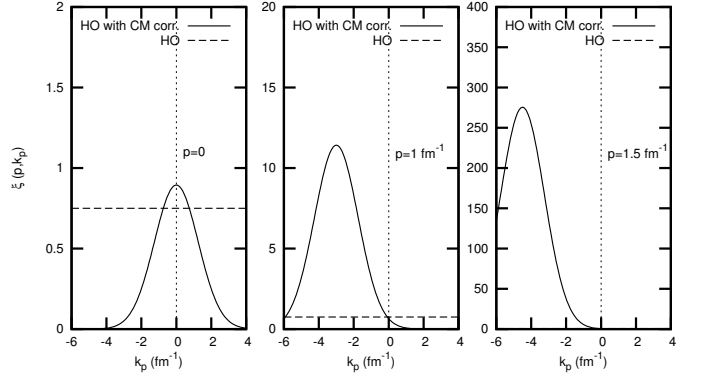$$\xi(\mathbf{p}, \mathbf{k}) \equiv \eta^{[2]}(\mathbf{p}, \mathbf{k})/\eta(\mathbf{p})\eta(\mathbf{k}) \tag{74}$$

Fig. 2. The quantity ξ of ^{4}He for $\mathbf{p} \parallel \mathbf{k}$ and $p = 0, 1, 1.5\,\mathrm{fm}^{-1}$, as a function of k_p, where $\mathbf{k} = k_p\hat{p}$, in the HOM model. Full line: with CM motion effects taken into account within the EST approach, eq. (76) for $b = b_{CM}$; dashed line: in the simple HOM, eq. (75), refs. [11,31].

was introduced, as a measure of correlations of statistical and dynamical origin as well as of finite-size effects. In the complete absence of correlations, ξ should be equal to $1 - 1/A$. In the case of the infinitely extended ideal Fermi gas ξ is defined for $p, k \leq p_F$, where p_F is the Fermi momentum, and if $\mathbf{p} \neq \mathbf{k}$, $\xi = 1$ (note that $A \to \infty$), while for $\mathbf{p} = \mathbf{k}$, $\xi = 1 - 1/\nu$ (ν is the degeneracy of the particle states). Even for the finite non-interacting fermion system, $\xi = 1 - 1/\nu$ if $\mathbf{p} = \mathbf{k}$ (because $\eta^{[2]}(\mathbf{p}, \mathbf{k}) = \eta(\mathbf{p})\eta(\mathbf{k}) - \frac{1}{\nu}|\eta_1(\mathbf{p}, \mathbf{k})|^2$ holds). Deviations of $\xi(\mathbf{p}, \mathbf{p})$ from this value show the effect of other-than-statistical correlations. For $\mathbf{p} \neq \mathbf{k}$, deviations from $1 - 1/A$ is a measure of statistical and (or) dynamical correlations in a system of finite size. In addition, in the case of self-bound systems, deviations from this value account for the correlation due to the fixed center of mass of the system.

The system of ^{4}He in the simple HOM is a special case for which $\nu = A$ and therefore

$$\xi = 1 - 1/\nu = 0.75 \tag{75}$$

for all $\mathbf{p}$ and $\mathbf{k}$. The same does not hold after fixing the center of mass. Then we have

$$\begin{aligned}\xi_{\mathrm{EST}}(\mathbf{p}, \mathbf{k}) &= \eta^{[2]}_{\mathrm{EST}}(\mathbf{p}, \mathbf{k})/\eta_{\mathrm{EST}}(\mathbf{p})\eta_{\mathrm{EST}}(\mathbf{k}) \\ &= 0.89493 \mathrm{e}^{-\frac{1}{6}p^2 b^2} \mathrm{e}^{-\frac{1}{6}k^2 b^2} \mathrm{e}^{-\mathbf{p}\cdot\mathbf{k}b^2}. \end{aligned} \tag{76}$$

In fig. 2, we plot ξ as a function of k_p, where $\mathbf{k} = k_p\hat{p}$, for $p = 0, 1, 1.5\,\mathrm{fm}^{-1}$. In fig. 3, $\log_{10} \xi$ is plotted as a function of $\cos\gamma$, where γ is the angle between $\mathbf{p}$ and $\mathbf{k}$. The full, long-dashed and short-dashed lines correspond to the $\xi_{\mathrm{EST}}(\mathbf{p}, \mathbf{k})$ of eq. (76) for $p = k = 1\,\mathrm{fm}^{-1}$, for $p = 1$, $k = 4\,\mathrm{fm}^{-1}$ and for $p = k = 4\,\mathrm{fm}^{-1}$, respectively, and the dashed lines to $\xi(\mathbf{p}, \mathbf{k}) = 0.75$, eq. (75). The effect of CM correlations is important. The EST TBMD favors momenta of opposite directions, as compared to the product of the two OBDM, while in forward angles ξ_{EST} is significantly reduced. In refs. [11,31] the effect of short-range correlations (SRC) was investigated by including in the simple HOM Jastrow-type correlations in the calculation of TBMD of the ^{4}He nucleus using the lowest-order
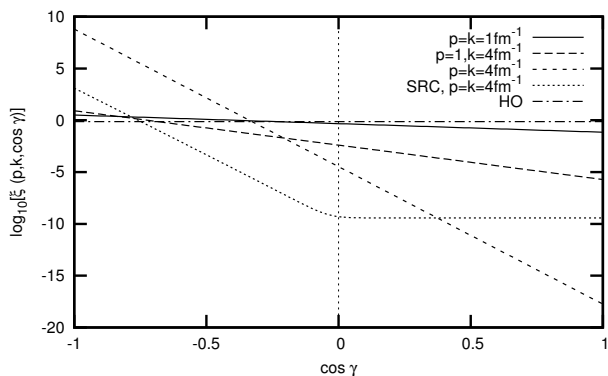
Fig. 3. The quantity $\log_{10} \xi$ of ^{4}He for the indicated values of p and k as a function of $\cos \gamma$, where γ is the angle between **p** and **k**, in the HOM model. Full, long-dashed, short-dashed lines: with CM motion effects taken into account within the EST approach, eq. (76) for $b = b_{CM}$; dotted line: including short-range correlations within the LOA [11], without CM motion effects taken into account; dot-dashed line: in the simple HOM, eq. (75) (result independent of p, k and γ).

approximation of ref. [32]. Significant deviations from the independent-particle picture were found for large values of p and k close to $\gamma = 180°$ and $0°$. In fig. 3 the corresponding quantity ξ for the case $p = k = 4\,\mathrm{fm}^{-1}$ is plotted in logarithmic scale for comparison. It is anticipated that within the EST approach additional corrections due to SRC will appear mainly at high values of p and/or k and that they will be larger when **p** and **k** are antiparallel.

6 Summary and conclusions

The intrinsic one-body and two-body density matrices in coordinate space and corresponding Fourier transforms in momentum space have been studied for a nucleus (a non-relativistic system) that consists of A nucleons (particles). We have seen how these quantities of primary concern can be expressed through expectation values of the A-particle multiplicative operators $\rho_{\mathrm{int}}^{[1]}$ and $\rho_{\mathrm{int}}^{[2]}$ sandwiched between intrinsic nuclear states. Our consideration is translationally invariant since the operators depend on the relative coordinates and momenta (Jacobi variables). To avoid a cumbersome multiple integration, we have developed an algebraic technique based upon the Cartesian representation, in which the Jacobi variables are the linear combinations of the creation and destruction operators for oscillator quanta in the three different space directions. In the framework of the subsequent operations the normal ordering of the operators involved in $\rho_{\mathrm{int}}^{[1]}$ and $\rho_{\mathrm{int}}^{[2]}$ plays a central role in getting both the general results and the working formulae.

The Cartesian representation is convenient and allows us to find simple links between the relevant distributions via the generating functions constructed here. In particular, the OBMD $\eta(\mathbf{p})$ and the elastic FF $F(q)$ can be deduced from one and the same generating function by changing the values of its arguments.

In the course of such a procedure the so-called Tassie-Barker factors stem directly from the intrinsic operators (not the WFs). One should emphasize that these factors (different for different distributions) occur here reflecting the translationally invariant structure of the corresponding intrinsic operators. Each of them is a Gaussian whose behavior in the space of variables is governed by the size parameter r_0 (or its reciprocal p_0) and the particle number A for a given system, but it does not depend upon the choice of the g.s. WF. The latter can be a simple Slater determinant, include SRC or not, be CM-corrected or not, etc. In practical calculations, such WFs (in particular, the reference WF $|\Psi\rangle$ in eq. (12)) are often expanded in the convenient HO basis functions. Therefore, in order to exploit all the power of the HO algebra when manipulating the intrinsic operators, it is pertinent to set the working r_0-value equal to the respective "optimum" value of the oscillator parameter.

In order to realize all the general results, obtained above for the intrinsic density matrices and related distributions, the intrinsic wave functions of the nuclear ground state have been constructed using the prescription by Ernst, Shakin and Thaler, that leads to the so-called fixed-CM approximation. In this connection, we have demonstrated how one can unify the different approximate recipes of restoring the TI, if one starts with one of them, e.g., with the EST projection operator. As a specific example, analytic expressions for the intrinsic OBMD $\eta(\mathbf{p})$ and TBMD $\eta^{[2]}(\mathbf{p}, \mathbf{k})$ of the ^{4}He nucleus have been derived within the context of the independent particle shell model, using harmonic-oscillator wave functions. When CM corrections are taken into account, the OBMD and TBMD are simultaneously shrunk with respect to the nontranslationally invariant counterparts. In addition, the CM correlation introduces in the case of the TBMD a dependence on the angle between **p** and **k**. A shift of its peak for $p \neq 0$ in favor of opposite momenta and significant deviations for large values of p and k and angles close to $180°$ are observed. The above calculation is relevant to the current experimental study of two-nucleon knock out off He isotopes. For instance, we see similar behavior as the one found in the recent experimental study of the TBMD in ^{3}He [5].

Of course, when increasing the momenta transferred to a residual system (in particular, under kinematic conditions where they are getting comparable with the nucleon mass) the corrections to TI breaking should be considered along with incorporating relativistic effects such as the Lorentz contraction of nuclear WFs and a specific velocity dependence of nuclear forces. The latter is typical of relativistic one-particle-exchange models (see survey [48] and references therein), where, for instance, the nucleon-nucleon quasipotential, including recoil effects, is essentially nonlocal and prevents, in contrast to the nonrelativistic case, the separation of the CM motion of the relativistic system (nucleus) from its internal motion. In other words, the corresponding four-momentum eigenstates cannot be factorized as a product of independent CM and intrinsic components. In this context, note the very instructive work of ref. [49]. Taking into account this distinctive

feature of relativistic quantum mechanics, it is difficult *a priori* to say to what extent the approach developed here could be helpful in a covariant description of composite systems. Nevertheless, an encouraging example is found in ref. [50], where the fixed-CM approximation combined with an appropriate Lorentz contraction has been used for calculations of the nucleon electromagnetic FFs in the cloudy bag model with CM and recoil corrections.

Using the techniques presented in this paper, one could calculate the intrinsic TBMD of other than ^{4}He $Z = N, \ell$-closed nuclei within the context of the harmonic-oscillator model. In addition, within the present framework one could investigate the effects of short-range correlations on the intrinsic TBMD of ^{4}He and other ℓ-closed nuclei, which are expected to be sizable for $p, k \geq p_F$, by introducing Jastrow-type correlations. Also, other intrinsic two-body quantities could be evaluated within the above general formalism (including other non-relativistic systems). Finally, the approach developed here may be helpful when evaluating the intrinsic one-body, two-body and more complicated density matrices in the HOM and in other independent-particle models (*e.g.*, with single-particle wave functions of a potential well with finite depth, as was shown in ref. [41]).

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